

Polynomial Equations and Graphs

Definition:

A **polynomial function** is any function that can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

The **leading term** of the polynomial above is $a_n x^n$

The leading coefficient of the polynomial above is a_n

The degree of the polynomial above is n .

There are many forms a given polynomial can be written in, but two common forms are expanded form and factored form.

$x^3 - 2x^2 - 5x + 6$ is the same polynomial as $(x - 1)(x - 3)(x + 2)$

Zeros of a function:

-If $f(k) = 0$ then k is said to be a **zero of the function f** .

-If k is an input value of the function f that returns an output of 0 then c is a **zero of the function f** .

-If f contains the point $(k, 0)$ then c is a **zero of the function f** .

Verify that 3 is a zero of the function $f(x) = 2x^3 - 5x^2 - 9$

If k is a zero of the function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ then we also say that k is a **root** or a **solution** of the equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$.

Verify that $x = 2 + i$ is a solution to the equation $x^2 = 4x - 5$

End Behaviors for the Graph of a Polynomial:

Consider the value of each term of the polynomial as x approaches infinity.

$$f(x) = -2x^3 + 4x^2 + 5x + 2$$

$$f(1) = -2 + 4 + 5 + 2 = 9$$

$$f(10) = -2000 + 400 + 50 + 2 = -1548$$

$$f(100) = -2000000 + 40000 + 500 + 2 = -1959498$$

Consider the value of each term of the polynomial as x approaches negative infinity.

$$f(x) = -2x^3 + 4x^2 + 5x + 2$$

$$f(-1) = 2 + 4 - 5 + 2 = 3$$

$$f(-10) = 2000 + 400 - 50 + 2 = 2352$$

$$f(-100) = 2000000 + 40000 - 500 + 2 = 2039502$$

To determine the right-end behavior of a polynomial function you need to determine if the leading term of the polynomial is positive or negative when x is positive.

To determine the left-end behavior of a polynomial function you need to determine if the leading term of the polynomial is positive or negative when x is negative.

Determine the end behavior for the following polynomial functions:

$$f(x) = -2x^4 - 3x^3 + 2x + 1$$

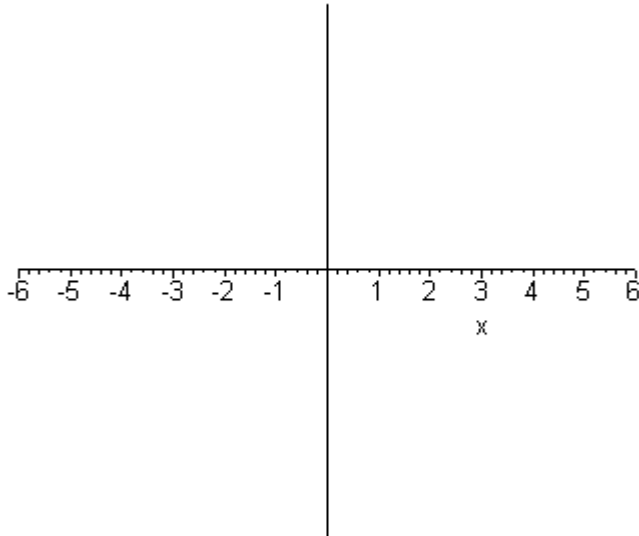
$$p(x) = 4x^5 + 7x^4 - 3x^3 + 2x^2 - 10x + 4$$

$$h(x) = -\frac{2}{3}x^7 + 3x^4 - 2x^2 + 11x - 4$$

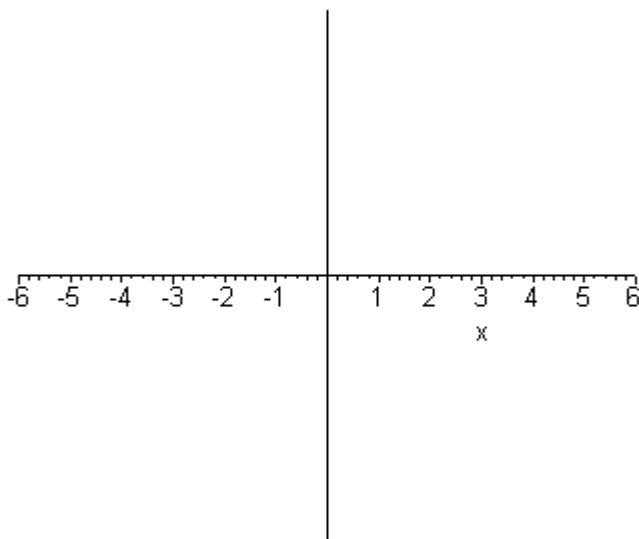
Examples:

Determine the end behaviors for the polynomial functions below and then draw the graphs of the following polynomials:

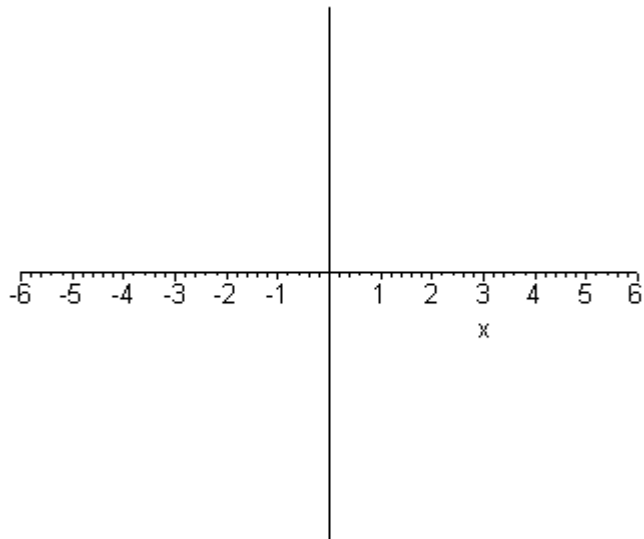
$$f(x) = 2(x - 2)(x + 1)^2$$



$$h(x) = -(x + 1)^2(x - 1)^2$$



$$p(x) = x^3 - 4x$$



Solve the inequality $x^3 > 4x$

Solve the inequality $2(x - 2)(x + 1)^2 > 0$